MECH 691: Convex Optimization

Project Proposal

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# **Motivation and Definition:**

Trajectory planning is a common problem in robotics, whether wheeled robots or unmanned aerial robots. According to [2], trajectory planning is the problem of creating motion from one point to another while avoiding collisions. Collisions can be defined as an event in which the robot gets into contact with any other object in its environment. As indicated by [2], trajectory planning is an NP-hard problem. According to [1] , NP-hard problems are problems whose algorithm is nondeterministic, polynomial time problem, meaning that their solution grows exponentially with problem size. Algorithms for tackling trajectory planning vary in discipline [3] and approaches and range from Node Based Optimal Algorithms, like A\* search or Dijkstra’s algorithm, to Bio-inspired Algorithms like Genetic Algorithms and Memetic Algorithms. In our project, we will try to approach this problem from a convex optimization perspective, aiming for multiple trajectory planning, with different physical constraints.

# **Literature Review:**

Different approaches to tackling the optimal trajectory planning problem have been researched. Some [4] approach the problem from a nonlinear programming approach using a Gauss pseudospectral method (GPM) as a solution while feasibly meeting the constraints. Other techniques prefer more classical approaches such as [5] A\* search algorithms that utilize grids to process the trajectory, or Dijkstra’s algorithm [6] which makes use of digital elevation models and heuristics combined with collision verification to tackle the problem in linear computational time O(np ), where nP is the number of verification steps.

In one of the candidate papers [7], the authors leverage convexification of constraints paired with physical models of K drones to draw the optimal trajectory for these K drones simultaneously. The authors also leverage an approach called receding horizon optimization (RHO), which basically iteratively solves the optimization problem, in order to allow for faster solving, making it more feasible for embedded trajectory planning. The authors also incorporate constraints relating to the physical (max,min speed and acceleration) of the K drones as well as obstacle avoidance constraints, to achieve complete trajectory planning with collision avoidance.

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# **Project Approach Plan:**

## RHO-based Convex Optimization

The first step in reproducing this paper would be to solve the static problem of trajectory planning for a single drone. The follow up step to this approach is characterized by incorporating obstacle constraints and modelling them into our constraints. Accordingly, we can further this problem by introducing multiple drones into the equation to generalize to swarms of drones as done in the paper. Finally, we will recreate the RHO approach suggested in the paper to enable a faster solver to the problem.

## Multi-robot Navigation in Formation Via Sequential Convex Programming:

For the multi-robot case, the task is split into two different optimization problems. The first component revolves around obtaining the locally optimal formation for the group of robots, while the second aspect requires motion planning around each robot to guarantee that no collisions occur when the group makes a transition in formation.

The local formation problem requires minimizing the weighted sum of the deviation with respect to the formation’s goal position (as seen by the centroid of the pack), total size, and rotation. The constraints, in this case, include the formation existing in the collision-free convex polytope that is directed towards the objective position (which must be computed), distances between the individual robots, and the rotation of the formation. This problem is solved by sequential quadratic programming using SNOPT, a non-linear solver.

After that, the local planning for the individual bots should be optimized to avoid crashes/collisions. To do so, the largest distance travelled for one robot should be minimized while taking into consideration a minimum deviation in the velocity and making sure that the motion is with the convex polytope (the set of allowable positions).[8]

## Generation of Collision-free Trajectories for a Quadrocopter Fleet: A Sequential Convex Programming Approach

The paper proposes an algorithm, which is based on sequential convex programming (SCP), that generates collision- free trajectory planning for multiple vehicles in 3D space. The objective is to transition from an initial to a final state, each consisting of the position, velocity and acceleration of each vehicle. In addition, the vehicles must maintain a minimum distance between each other and satisfy other trajectory constraints. This approach can be applied to any flying vehicle, but it is applied to quadrocopters in this paper[9].

The trajectory planning problem is an optimization problem where the objective function is the sum of the total thrust at each time step and the optimization variable is the vector consisting of the vehicles’ accelerations at each time step. This objective function can be expressed as a quadratic function. The problem consists of affine equality and inequality constraints. The equality constraints are due to the position, velocity and acceleration at the initial and final states. The inequality constraints are due to the position and velocity boundaries in limited working spaces. Other than that, the acceleration is limited by a maximum and minimum thrust value and the jerk, the third derivative of position, is also bounded in order to obtain feasible trajectories. The problem also consists of non-convex constraints which arise from the vehicles not colliding during transition. Then, the non-convex constraints are replaced with convex approximations. The collision avoidance constraint is linearized around the previous solution using first order Taylor expansion. This results in a quadratic program since the objective function is quadratic and the constraints are affine. The convex problem is solved iteratively depending on the initial guess and using sequential convex programming. This approach resulted in smooth trajectories that allow for takeoff and landing safely from arbitrary locations[9].

# **Bonus Steps:**

## RHO-based Convex Optimization

To further the RHO-based paper’s development, we will hope to introduce this solver as a real-time tool to enable the user to experiment around with different obstacles and different obstacle sizes, as well as different drone start and stop locations.

Additionally, we hope to add physical constraints of the different drones to enable collision avoidance between the drones themselves.

# **Expected Problems:**

## RHO-based Convex Optimization

Potential problems that we might face while trying to replicate this paper is the lack of a physical model of the plant that might reference constraints faced in real life scenarios. While this paper does mention Model Predictive Control as an approach to enabling faster solving of the problem, it lacks robust dynamical models (apart from acceleration, speed, and position relationships) of our drones which might affect our solution.

# **References:**

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